Complementarity and Afshar's experiment

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In this article we criticize the experiment realized by S. Afshar [Proc. SPIE 5866, 229-244 (July 2005)]. We analyze Bohr's complementarity and show that the interpretation proposed by Afshar is misleading.

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In an article recently submitted for publication [1] S. Afshar claims to have successfully realized an experiment disproving the complementarity principle of N. Bohr [2, 3, 4, 5]. Obviously this result, if verified, would constitute a serious attack against the orthodox interpretation of quantum mechanics (known as the Copenhagen interpretation). However the aim of this comment is to criticize briefly the result obtained by Afshar and to show that the interpretation presented in [1] is completely misleading.

In the following we will not consider the exact set up used in [1] but a slightly modified version. However since we are only interested in the general principle this idealized version of [1] , which is sketched in Fig. 1, is completely sufficient for our purpose. As preliminary analysis, we consider the Young double-pinhole experiment in the case of a single photon wave train impinging normally on the apertures plane. The photon state immediately behind the holes evolves into a sum of two diffracted waves: $\Psi(\mathbf{x}) = \Psi_A(\mathbf{x}) + \Psi_B(\mathbf{x})$, where $\Psi_{A,B}$ are single aperture wave functions reducing to two narrow peaks located at $\mathbf{x}_{B,A} = \pm d/2\hat{\mathbf{x}}$ in the apertures plane. The two spherical waves don't need to have necessary the same amplitude and we note $\Psi(\mathbf{x}) \sim Af(\mathbf{x}-\mathbf{x}_A) + Bf(\mathbf{x}-\mathbf{x}_B)$ the photon wave function in the apertures plane. We have typically $f(\mathbf{x}) \sim \delta^2(\mathbf{x})$ i. e. a two dimensional Dirac function. Neglecting the overlap between the two waves in the holes the intensity $I(\mathbf{x}) \propto |\Psi(\mathbf{x})|^2$ equals approximately $|Af(\mathbf{x}-\mathbf{x}_A)|^2 + |Bf(\mathbf{x}-\mathbf{x}_B)|^2$. If we put a detection screen just behind the apertures plane we expect to see two narrow peaks of relative intensities $P_A = |A|^2/\left(|A|^2+|B|^2\right)$ and $P_B = |B|^2/\left(|A|^2+|B|^2\right)$. $P_{A,B}$ define then probabilities to record a photon in A or B. In this context we can define the distinguishability

$$D = |P_A - P_B| = ||A|^2 - |B|^2|/\left(|A|^2+|B|^2\right). \qquad (1)$$

The meaning of D is clear if we consider the case $P_A = P_B$ corresponding to $D = 0$. Indeed if we detect a photon in the far field of the aperture planes (i. e. in the plane where we expect to see fringes) we can not say from which hole cames from the photon: the two holes are indistinguishable. Oppositely if $P_B = 1 = D$ we are sure that the photon detected in the far field came from B and not from A. In order to obtain a mathematical formulation of wave-particle duality we must consider the interference pattern recorded in the far-field. To do that we can introduce a converging lens behind the apertures plane. The effect of this lens on the photon can be mathematically described by Fresnel's diffraction theory. We consider here a lens with a gaussian transmission $T(\mathbf{x}) = e^{-(\mathbf{x}/\sigma)^2/2}$. We omit the irrelevant z dependent factor in the wave function (the z axis being the optical axis) and we deduce

$$\Psi(\mathbf{X}) \propto Ae^{-k^2\alpha\left(\mathbf{X}/P'+\mathbf{x}_A/P\right)^2} + Be^{-k^2\alpha\left(\mathbf{X}/P'+\mathbf{x}_B/P\right)^2} \qquad (2)$$

for every point $\mathbf{X} = [X, Y], z$ located after the lens. We have $\alpha^{-1} = 2\left(1/\sigma^2 - ik\epsilon\right)$ (where $k = 2\pi/\lambda$ is the photon wave vector) and $\epsilon = 1/P + 1/P' - 1/f$ which depends on the focal length f, the distance P between the lens and the pinholes, and the distance P' between the lens and the observation plane. In the focal plane of the lens we obtain

$$\psi(\mathbf{X}) = N \cdot [Ae^{-ikP(\mathbf{X}/f+\mathbf{x}_A/P)^2/2} + Be^{-ikP(\mathbf{X}/f+\mathbf{x}_B/P)^2/2}], \qquad (3)$$

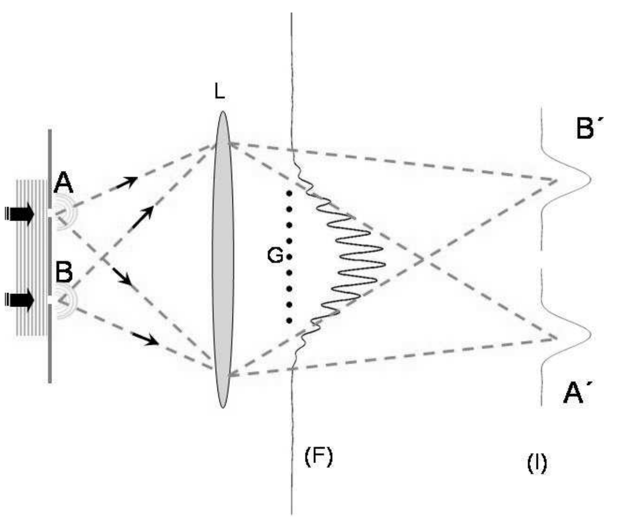

FIG. 1: A idealized version of Afshar's experiment. This experiment is very close from a gedanken experiment proposed originally by Wheeler[5].

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with the approximation $1/\sigma^2 \ll k/P$. The intensity is thus $I(\mathbf{X}) \propto 1 + V\cos(kd\hat{\mathbf{x}}\cdot\mathbf{X}/f + \phi)$ where the visibility V and the phase shift ϕ are defined by

$$V = 2|A||B|/\left(|A|^2+|B|^2\right)$$
$$\phi = \arg(A) - \arg(B). \quad (4)$$

Such visibility is equivalently defined by the relation $V = (I_{max} - I_{min})/(I_{max} + I_{min})$. It should be noted that we could like Afshar observe fringes in front of the lens where the intensity reads $I(\mathbf{X}) \propto 1 + V\cos(kd\hat{\mathbf{x}}\cdot\mathbf{X}/p + \phi)$ and V, ϕ are the same than previously. If $p \gg f$ it is indeed easier to observe the fringes in front of the lens.

It exists a strong relation between V and D since we have

$$D^2 + V^2 = 1 \quad \text{(duality relation[6])}. \quad (5)$$

It means that the observation of a perfect interference pattern with unit visibility $V = 1$ implies a total indistinguishability $D = 0$. Oppositely if we can predict from which hole the photon came from (i. e. $D = 1$) then we can't have interference: i. e. $V = 0$. This is already a formulation of the principle of complementarity. In this example this principle can be enounced in the following form: since we can not absorb a photon twice we must select between either observing the photon in the interference plane or in the aperture plane. If, with both apertures open and $|A| = |B|$, we detect several photons in the interference plane, then we will (by statistical accumulation) record an interference pattern with unit visibility. However in counterpart we can not build up, *by using the same events*, the statistical distribution of particles in the aperture plane because we have no information for that. The only case in which we can build the two patterns with the same particles is if we consider a single aperture (i. e. $D = 1$). But in that case the fringes visibility obviously equals zero. We considered here a simple experiment but the validity of this duality relation is in fact much more general[6]. Eq. 5 is applicable in cases involving quantum entanglement with a "which-path" detector. The aim of this comment is however not to discuss the concept of complementarity in its general sense and we refer to the original literature for that purpose [2, 6, 8].

The previous canonical example was slightly modified by S. Afshar. In his experiment Afshar considered actually what happens if we record the photon in the image plane of the lens (where $\epsilon = 0$ from geometrical optics). We have

$$\Psi(\mathbf{X}) =\propto Ae^{-k^2\sigma^2\left(\mathbf{X}/P' + \mathbf{x}_A/P\right)^2/2}$$
$$+Be^{-k^2\sigma^2\left(\mathbf{X}/P' + \mathbf{x}_B/P\right)^2/2} \quad (6)$$

which give us (in the limit $k\sigma d/P \ll 1$) two well separated gaussian spots centered on the geometrical image points A' and B' of the pinholes A and B. The intensity is then approximatively given by

$$I(\mathbf{X}) \propto |A|^2 e^{-k^2\sigma^2\left(\mathbf{X}/P' + \mathbf{x}_A/P\right)^2}$$
$$+|B|^2 e^{-k^2\sigma^2\left(\mathbf{X}/P' + \mathbf{x}_B/P\right)^2}. \quad (7)$$

At the limit of the infinite lens we obtain two dirac pics $I(\mathbf{X}) \propto |A|^2\delta^2(\mathbf{X}/P' + \mathbf{x}_A/P) + |B|^2\delta^2(\mathbf{X}/P' + \mathbf{x}_B/P)$. Eq. 7 implies that the number of particle detected at the center of the image A' (respectively B') is directly proportional to the number of particles potentially detectable in A (respectively B). We can as previously case introduce the same relative intensity $P_{A,B}$ and define the distinguishability D in an identical way. Obviously the duality relation Eq. 5 is still valid.

At that point a comment is necessary. Indeed the usual formulation of Bohr's principle says that any devices capable of determining the path taken by a particle trough the double-aperture must destroy the interference. The term path very often used in discussions concerning the wave-particle duality is however ambiguous and non univocally defined. More precisely it is not obvious in the present experiment (with both pinholes open) that a photon coming, say from A, should necessarily finish its journey in A'. This axiom is strongly dependent of the ontological model of hidden variable considered. In particular the ontological model of Bohm when generalized to photons leads to difficulties (see for example [7, 9]). For this reason we must limit references to the word "path" in quantum mechanics. In fact we can enounce Bohr's principle independently of any ontological hypothesis. Considering the previous double-pinhole example we can say that it is impossible by using the same events to build up the statistical distribution associated with two complementary observable of a same quantum state. This is clear since the wave function in the interference plane is the fourier transform of the wave function in the aperture plane. Recording photons in the image plane of the lens change nothing to the analysis: if we detect events in the image A' and B' we can build up the statistical distribution of photons in the aperture plane. However we can not build up the interference pattern. To do that we need to know in which point would be detected each individual events in the focal plane. However this exclude to use the same photons for building up the image spots A' and B' since a photn can not be absorbed twice. Once again this statement is independent of any ontological models considered: it is valid if we think in term of Bohm's trajectory, spontaneous collapse or whatever you want if this agrees with quantum mechanics.

In the same context it is important to remark that Afshar's use of Eq. 5 is misleading and confusing. Indeed he remarked that if we consider only one of the image we can define a population of photons for which $D' = 1$ and not $D = 0$. This is correct but then we can not use the relation $D'^2 + V^2 = 1$ since we refer to two different ensembles of particles (only half of the events are detected in A' or B'). If we record photons in the fo-

cal plane of the lens (i. e. in the interference plane) we can not attribute a label to photons saying in which of the two images A' or B' it will or would finish its journey (if he was not already recorded in the focal plane). Afshar didn't respected these limitations and found that $D'^2 + V^2 = 1 + 1 = 2$. This is mathematically true but this is not the duality relation and can not be considered as a formulation of Bohr's principle.

In the last steps of his experiment Afshar introduced absorbing wires at the interference minima locations. If the wires are sufficiently thin we don't expect an effect on the image spots A' and B' when both pinholes are open. In his particular set up Afshar observed actually a small reduction of the intensity of $R \sim 0.1\%$ due to absorbtion by the wires. This intensity reduction is much smaller than the reduction $R \ \sim 6\%$ expected for a uniform flow of energy in the interference plane (this can be experimentally tested by opening only one of the two pinholes). This is clearly an indication that interference occurs in the focal plane of the lens. Afshar then claims that he recorded both the statistical distribution of particles in the image plane and the interference pattern.

A first and intuitive critic against this interpretation is that the grid of wires acts like a grating disturbing the motion of the photons. It is indeed visible with the grid in place and by successively opening and closing one of the two pinhole, say A (B being always open), that the shape of the spot A' is modified [1]. This obviously proves that a contribution coming from B is going in A' when both holes are open. One could then argue that the which path information is lost during this interaction with the grating. A simple way to see this effect is to consider a simple wire or diffracting object located in the Fourier plane (i. e. the focal plane) of the lens. This object induce a scattered field which can be in principle written $\simeq \alpha\Psi(\mathbf{x}_0)e^{ik(\mathbf{x}_0-\mathbf{x})^2/(2R)}$ where $R = P' - f$ is the distance separating the focal plane from the the image plane, $\mathbf{x}_0$ the x-y coordinates of the diffracting object in the focal plane, and $\mathbf{x}$ the x-y coordinates in the image plane. Additionally we introduce an effective polarizability α of the diffracting object. The total intensity in the image plane is thus

$$|\alpha\Psi(\mathbf{x}_0)|^2 + |\Psi_A(\mathbf{x}) + \Psi_B(\mathbf{x})|^2 + \\ (\Psi_A(\mathbf{x}) + \Psi_B(\mathbf{x})) \cdot (\alpha\Psi(\mathbf{x}_0) + \alpha^*\Psi^*(\mathbf{x}_0)), \tag{8}$$

where $\Psi_A(\mathbf{x}) + \Psi_B(\mathbf{x})$ is the field given by Eq. 6. In the ideal case of a infinite lens we see that not only the scattering affect the value of the intensity at the maximum of the peaks A' and B' but that in addition there is a background signal $|\alpha\Psi(\mathbf{x}_0)|^2$ everywhere in the image plane. These kind of objections have been presented in particular by W. G. Unruh (private communications and internet discussions) but I don't think personally that this is a fundamental problem. Indeed it can be observed that in the configuration used by Afshar (with both holes open and with the grid) the effect of the wires is practically zero since the field is practically zero on the cross-section region of the wire. There is then no scattering light in this configuration which means that the wire don't disturb significantly the propagation. It can be even noted that in principle one can consider objects like atoms which are in very good approximation punctual. Using Eq. 8 with such object shows that the field $\Psi(\mathbf{x}_0)$ at the atoms location equal zero when both hole A and B are open. This means a perfect and ideal compensation of the waves reaching A' (or B') and resulting of the atoms excitation of waves coming from A and B. Naturally the result is different with a single aperture. However we can not compare the single and double pinhole experiment simply by comparing the intensity and by thinking in terms of trajectory like in classical mechanics. Indeed in classical mechanics a particle going through A (respectively B) does not care about the existence of B (respectively A). It is in that sense that we have the right to consider the sum of probability distributions in classical physics. However this is trivially not true in quantum physics. If we thus conclude (wrongly) that scattering in the single aperture experiment imply which path losses in the double-aperture experiment then we don't consider the problem of wave duality and this is necessarily a misleading assumption. The intuitive " classical " reasoning is consequently criticizable since it uses again the ambiguous notion of "path" or trajectory in the context of quantum theory based on the principle of wave superposition.
Recently Ole Steurnagel [10] analyzed numerically the effect of the grid on the waves coming from A and B and concluded like us that a wave contribution coming from A (respectively B) reaches necessarily B' (respectively A') since when we open both slit the initial pattern is practically recovered. However in [10] it is concluded that this means a loss in the which path information. This is again the same confusion: With optics one can consider two configurations (here two single-aperture experiments) and add the waves to obtain a new physical configuration. However this wave-based reasoning doesn't mean that we have lost which path information when both holes are open and the grid is present since the concept of path is not univocal in this case. Bohr pertinently observed that the concept of trajectory is senseless for an experimentalist in the double hole experiment since we can never observe the path without destroying fringes.

It should be finally noted in this context that in the Bohm interpretation the photon trajectories with both holes open are not significantly disturbed and this is in agreement with the weak reduction of $R \sim 0.1\%$. This model is non classical since it involves the quantum potential which is context dependent[11] and change strogly when we open or close one of the two holes. The Bohm model use the concept of trajectory but since these "real" paths are hidden and not observable this never contradict Bohr's statement.

The real problem in Afshar's interpretation comes from the fact that the interference pattern is not actually completely recorded. From the data of Afshar we would only

be able to build up the part of the interference pattern corresponding to the $R \sim 0.1\%$ of particles absorbed by the wires. This would allow us in principle to say how look the interference pattern close to the fringes minima. However for the rest of the photons actually detected in A' or B' we can not say anything. The situation is then completely identical to the one without wires. The problem is that Afshar accepts the apriori validity of wave optics. He supposes that the small value of $R \sim 0.1\%$ is sufficient to deduce the existence of fringes without measuring them. This is a logical inference in the context of classical optics but this is forbidden in quantum mechanics since we need different photons to observe both interference and images patterns. This is not a philosophical result but an experimental consequence of the quantum mechanics. It is interesting to remark that a reasoning similar to the one presented by Afshar leads to identical paradox when applied to a configuration without lens. Indeed in that case we have fringes in the far-field and we can deduce from the existence of the interference pattern the existence of the two coherent sources A, B since wave optics has no other way to justify fringes like that. Would you say that we deduced *experimentally* the existence of the source A and B? for sure not and this is the same in the Afshar experiment: We are not able to use the same particles to build up the different statistics in the two complementary planes. We can even make a parallel with the experiment of Afshar by suggesting an other simple experiment. Suppose that instead of having in the object plane of the lens two peaks well defined we have now as periodical sinusoidal field. Thus from Fourier optics we will deduce that in the back focal plane of the lens we have now two well localized peaks and that in the image plane we have (magnified) interference fringes. We have now completely the right to add in the back focal plane a screen with two apertures centered in the peaks position. If the holes are not too small this will not affect the propagation since the field is null on the wall of the screen. Now if we observe photons in the image plane we see fringes after statistical accumulation. However we can conclude that the probability of presence is equal to zero on all the screen in the focal plane since no photon are absorbed reflected and diffracted. Do we measure in that case the complete pattern in both complementary planes (back focal plane and image plane)? Again the answer is not since we don't measure what happens in the region of the apertures. It is only the complete measurement of the two complementary patterns with the same particles which could invoked to break the principle of complementarity. But this is impossible as already demonstrated by Bohr.

To conclude, in spite of Afshar's claim we still need two experiments in order to exploit the totality of the phenomenon. As pointed out originally by Bohr, we can not use information associated with *a same* photon event to rebuild in a statistical way (i.e. by *an accumulation of such events*) the two complementary distributions of photons in the image plane and in the interference plane. The hypothesis of Afshar that we only need some partial information concerning the interference pattern in order to reconstruct the complete interference is only based on the idea that the fringes already exist. The whole reasoning is circular and for this reason misleading.

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